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# TRIGONOMETRÍA

GRUPO PITÁGORAS

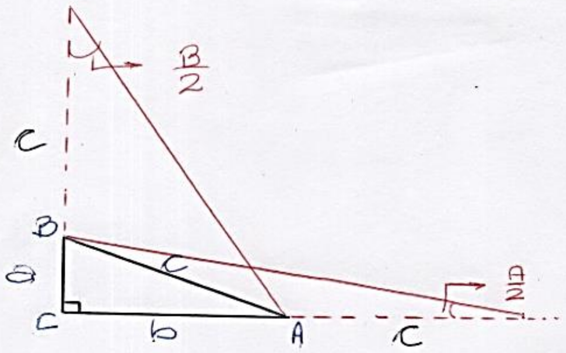
# Problema 5:

En un triangulo ABC (C=90°) se cumple que: $\text{Tan}A = \dfrac{2+\text{Cot}\frac{B}{2}}{4+\text{Cot}\frac{A}{2}}$

Calcule: $T = \dfrac{\text{Sec}^2 B - \text{Cot}A}{4+\text{Csc}^2 A}$

A) 1 B) $\frac{1}{2}$ C) $\frac{1}{3}$ D) 3 E) 2

5:



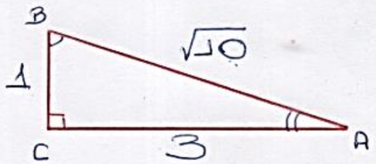

$$\tan A = \frac{2 + \cot\left(\frac{B}{2}\right)}{4 + \cot\left(\frac{A}{2}\right)}$$

$$\frac{a}{b} = \frac{2 + \left(\frac{c+a}{b}\right)}{4 + \left(\frac{b+c}{a}\right)}$$

$$4a + b + \cancel{c} = 2b + \cancel{c} + a$$

$$3a = 1b$$

$$\frac{a}{b} = \frac{1}{3} \longrightarrow$$




Calcular: $T = \dfrac{\sec^2 B - \cot A}{4 + \csc^2 A}$

$$T = \frac{(\sqrt{10})^2 - 3}{4 + (\sqrt{10})^2}$$

$$T = \frac{7}{14} \qquad \therefore \boxed{T = \frac{1}{2}}$$

CLAVE B

# Problema 6:

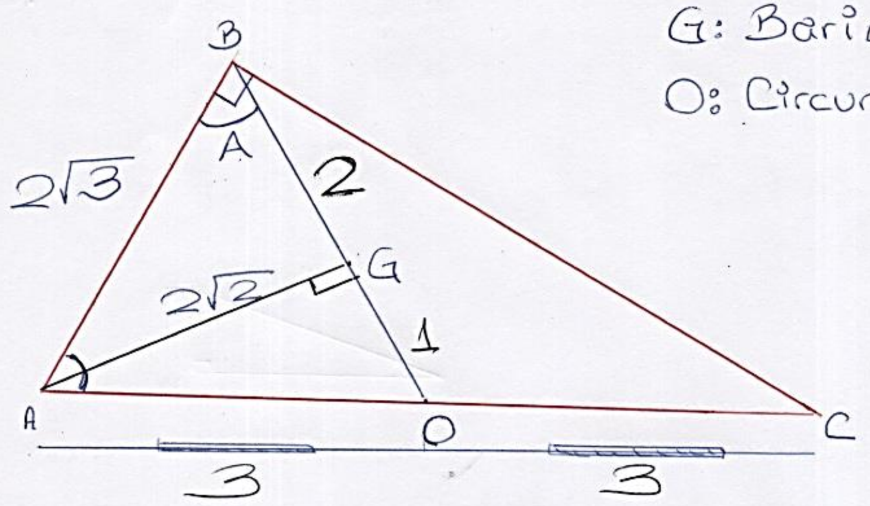
En un triangulo rectángulo ABC (B=90°) de baricentro G y circuncentro O, se tiene que $m\measuredangle AGO = 90°$.Calcule CscA

A) $\frac{\sqrt{2}}{2}$ B) $\frac{\sqrt{3}}{2}$ C) $\frac{\sqrt{5}}{2}$ D) 2 E) $\frac{\sqrt{6}}{2}$

6.

G: Baricentro

O: Circuncentro




- $BG = 2 \wedge GO = 1$
- T. Pitágoras: $\triangle AGO$: $3^2 = 1^2 + \overline{AG}^2$

  $\overline{AG} = 2\sqrt{2}$
- T. Pitágoras $\triangle AGB$: $\overline{AB}^2 = (2\sqrt{2})^2 + 2^2$

  $\overline{AB} = 2\sqrt{3}$

calcular: $\operatorname{Csc} A = \dfrac{\cancel{2}\sqrt{3}}{\cancel{2}\sqrt{2}}$

$\therefore \operatorname{Csc} A = \dfrac{\sqrt{6}}{2}$

CLAVE E

# Problema 7:

Si $E = \dfrac{x^2 - y^2 Sen20° + xy(Cos70° - 1)}{x^2 + y^2 Cos70° + xy(Sen20° + 1)}$

Calcular: $P = \dfrac{1+E}{1-E}$

A) $\dfrac{x}{y}$  B) $\dfrac{y}{x}$  C) $\dfrac{y}{2x}$  D) $\dfrac{2x}{y}$  E) $\dfrac{3y}{x}$

7.

$$E=\frac{x^2-y^2\operatorname{Sen}20^\circ+xy(\operatorname{Cos}70^\circ-1)}{x^2+y^2\operatorname{Cos}70^\circ+xy(\operatorname{Sen}20^\circ+1)}$$

$$\frac{E}{1}=\frac{x^2-y^2\operatorname{Sen}20^\circ+xy\operatorname{Sen}20^\circ-xy}{x^2+y^2\operatorname{Sen}20^\circ+xy\operatorname{Sen}20^\circ+xy}$$

P.R.P.:

$$\frac{1+E}{1-E}=\frac{2x^2+2xy\operatorname{Sen}20^\circ}{2y^2\operatorname{Sen}20^\circ+2xy}$$

$$P=\frac{x\,\cancel{(x+y\operatorname{Sen}20^\circ)}}{y\,\cancel{(y\operatorname{Sen}20^\circ+x)}}$$

$$\therefore\ P=\frac{x}{y}$$

CLAVE A

# Problema 8:

El perímetro de un triangulo rectángulo ABC (recto en A) es 1 m.

El equivalente de: $b^2\left(\frac{SecB+1}{SecB-1}\right)^{\frac{1}{2}}+c^2\left(\frac{SecC+1}{SecC-1}\right)^{\frac{1}{2}}$

en términos de "a" es :

A) $a^2-3a+1$

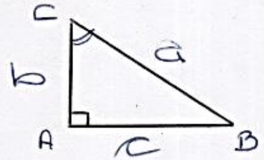
B) $a^2+3a+1$

C) $1+3a-a^2$

D) $1+a-a^2$

E) $1-a-a^2$

8.-




$$a^2=b^2+c^2$$
$$a^2-c^2=b^2$$
$$a^2-b^2=c^2$$

i) $a+b+c=1m \begin{cases} a+c=1-b \\ a+b=1-c \end{cases}$

$$b^2\cdot\left[\frac{\operatorname{Sec}B+1}{\operatorname{Sec}B-1}\right]^{\frac{1}{2}}+c^2\left[\frac{\operatorname{Sec}C+1}{\operatorname{Sec}C-1}\right]^{\frac{1}{2}}$$

$$b^2\cdot\sqrt{\frac{\frac{a}{c}+1}{\frac{a}{c}-1}}+c^2\cdot\sqrt{\frac{\frac{a}{b}+1}{\frac{a}{b}-1}}$$

$$b^2\cdot\sqrt{\frac{(a+c)}{(a-c)}\cdot\frac{(a+c)}{(a+c)}}+c^2\cdot\sqrt{\frac{(a+b)}{(a-b)}\cdot\frac{(a+b)}{(a+b)}}$$

$$b^{\cancel{2}}\cdot\frac{(a+c)}{\cancel{b}}+c^{\cancel{2}}\cdot\frac{(a+b)}{\cancel{c}}$$

$$b(1-b)+c(1-c)$$

$$b-b^2+c-c^2$$

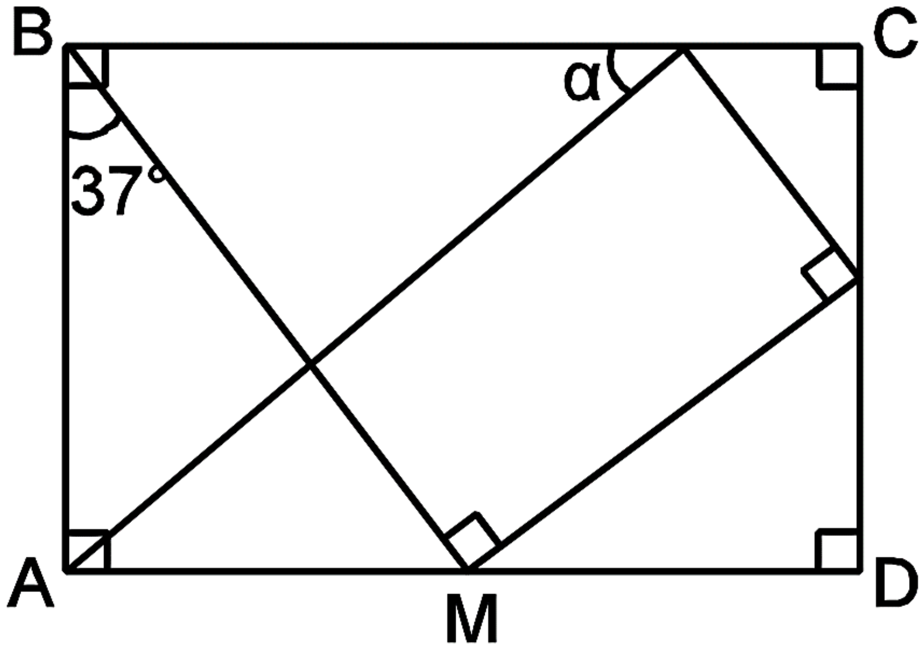
$$\underbrace{b+c}_{1-a}-(\underbrace{b^2+c^2}_{a^2})$$

$$1-a-a^2$$

CLAVE E

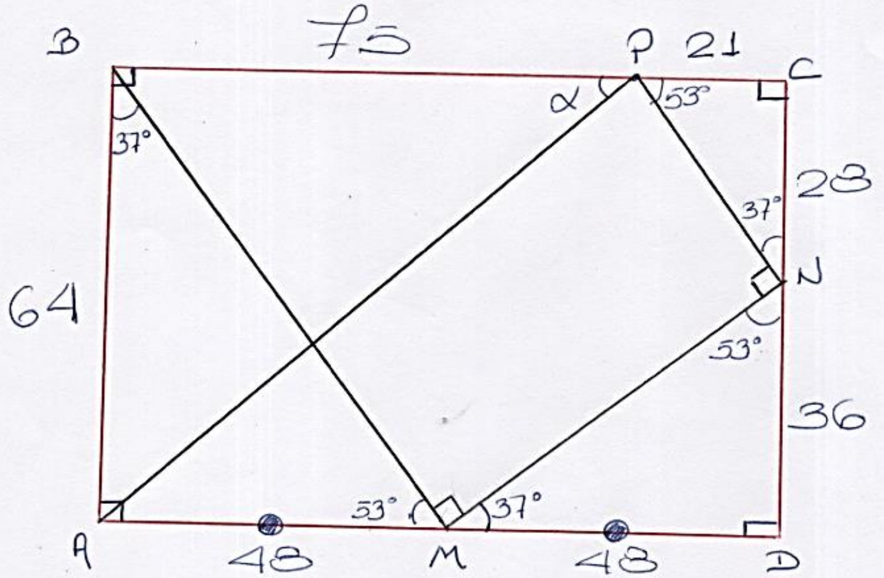
# Problema 9:

Calcule Tanα, si: AM=MD




A) $\frac{63}{151}$ B) $\frac{61}{70}$ C) $\frac{31}{4}$ D) $\frac{64}{75}$ E) $\frac{61}{23}$

9.-



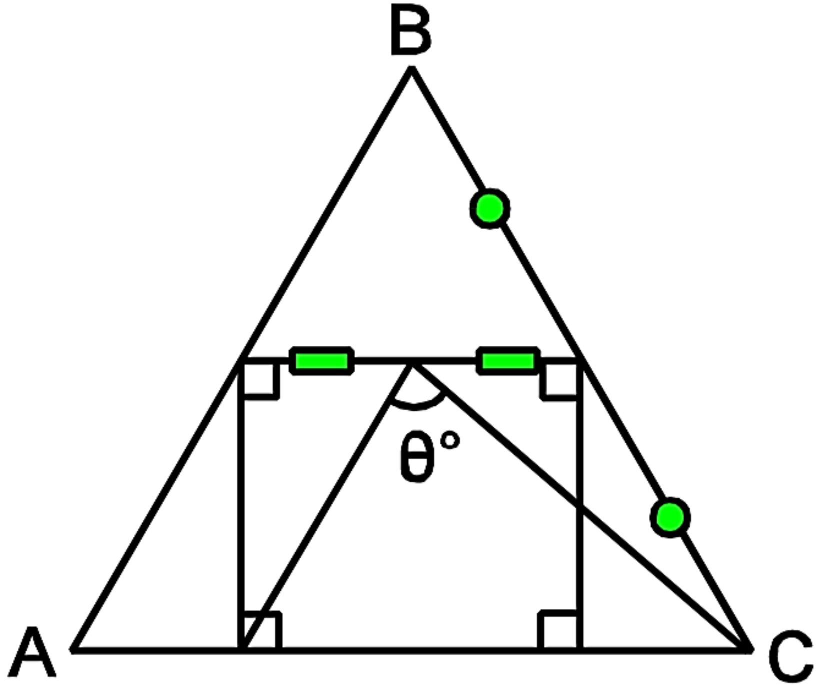

● $\begin{cases} \overset{\circ}{3} \\ \overset{\circ}{4} \end{cases}$   ◍ = $\overset{\circ}{12}$

$$\tan\alpha = \frac{64}{75}$$

CLAVE D

# Problema 10:

Del grafico adjunto, calcule Senθ siendo el triangulo ABC equilátero.




A) $\frac{3\sqrt{21}}{17}$

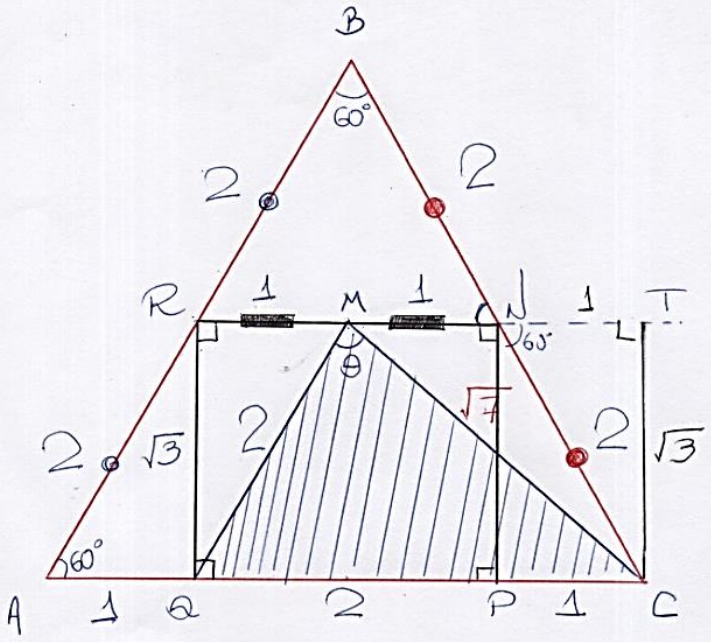
B) $\frac{\sqrt{21}}{14}$

C) $\frac{\sqrt{21}}{7}$

D) $\frac{3\sqrt{21}}{21}$

E) $\frac{3\sqrt{21}}{14}$

10.- $\Delta ABC$: Equilátero




T. Pitágoras: $\overline{MC}^2 = 2^2 + \sqrt{3}^2 \rightarrow \overline{MC} = \sqrt{7}$

$$S_{\Delta QMC} = \frac{2 \cdot \sqrt{7}}{\cancel{2}} \operatorname{Sen}\theta = \frac{3 \cdot \sqrt{3}}{\cancel{2}}$$

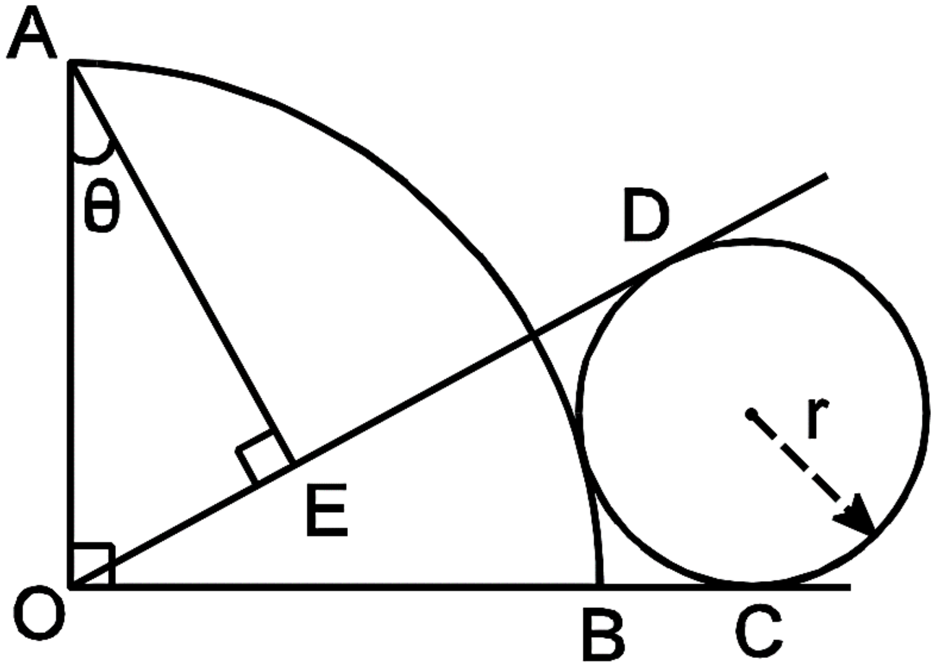
$$\operatorname{Sen}\theta = \frac{3\sqrt{3}}{2\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}}$$

$$\operatorname{Sen}\theta = \frac{3\sqrt{21}}{14}$$

CLAVE E

# Problema 11:

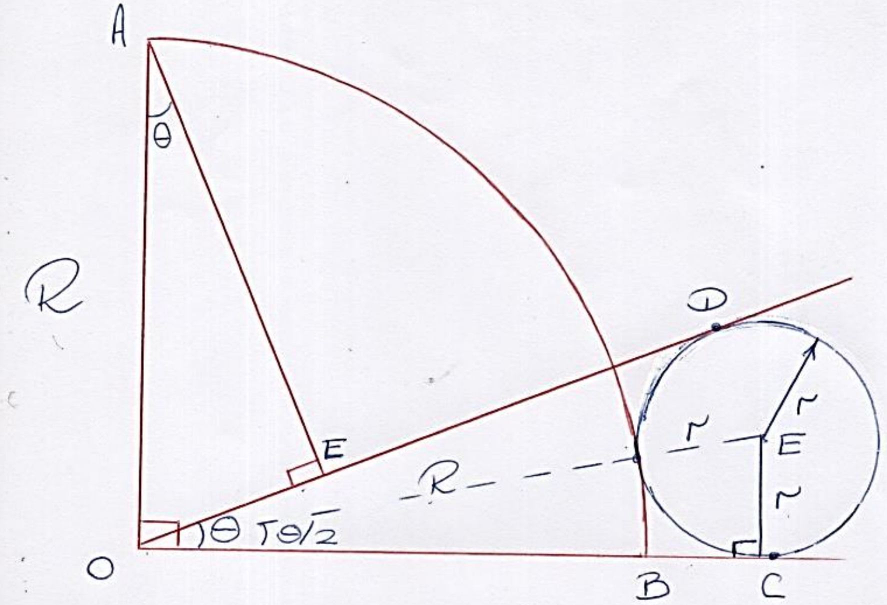
Dado un sector circular AOB, C y D son puntos de tangencia, calcular AE en términos de θ y r




A) $r\cos\theta\left(\csc\frac{\theta}{2}-1\right)$ B) $r\cos\theta\left(\csc\frac{\theta}{2}+1\right)$ C) $r\sin\theta\left(\sec\frac{\theta}{2}-1\right)$

D) $r\sin\theta\left(\sec\frac{\theta}{2}+1\right)$ E) $r\sin\frac{\theta}{2}\left(\sec\frac{\theta}{2}-1\right)$

11- $AE = ?$




$\triangle OCE$: $\overline{OE} = r\,Csc\left(\frac{\theta}{2}\right)$

$R + r = r\,Csc\left(\frac{\theta}{2}\right)$

$R = r\left(Csc\frac{\theta}{2} - 1\right)$

$\triangle OEA$:

$AE = R\,Cos\theta$

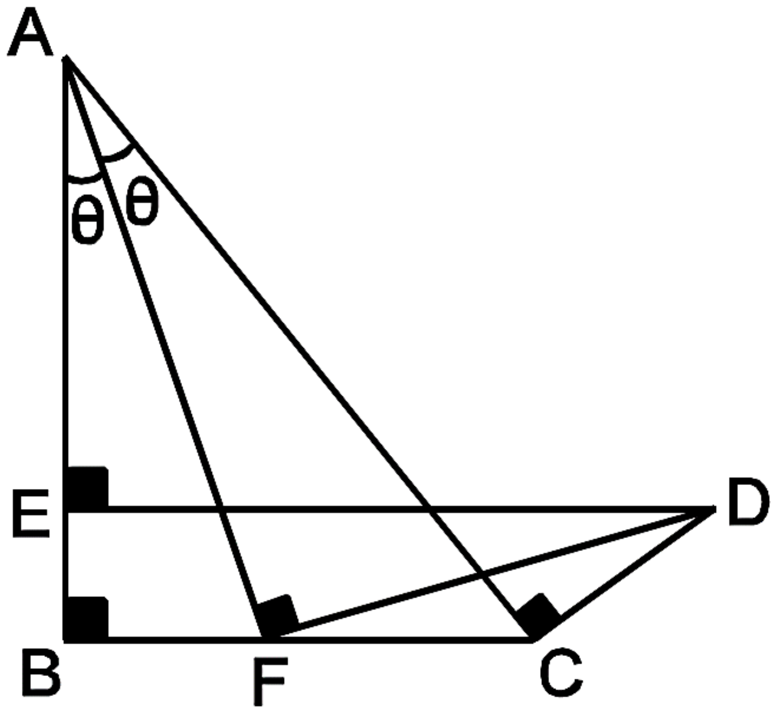
$\therefore\ AE = r\left(Csc\frac{\theta}{2} - 1\right)Cos\theta$

CLAVE A

# Problema 12:

En la figura: $\frac{\mathrm{ED}}{\mathrm{CF}} = \frac{146}{50}$.

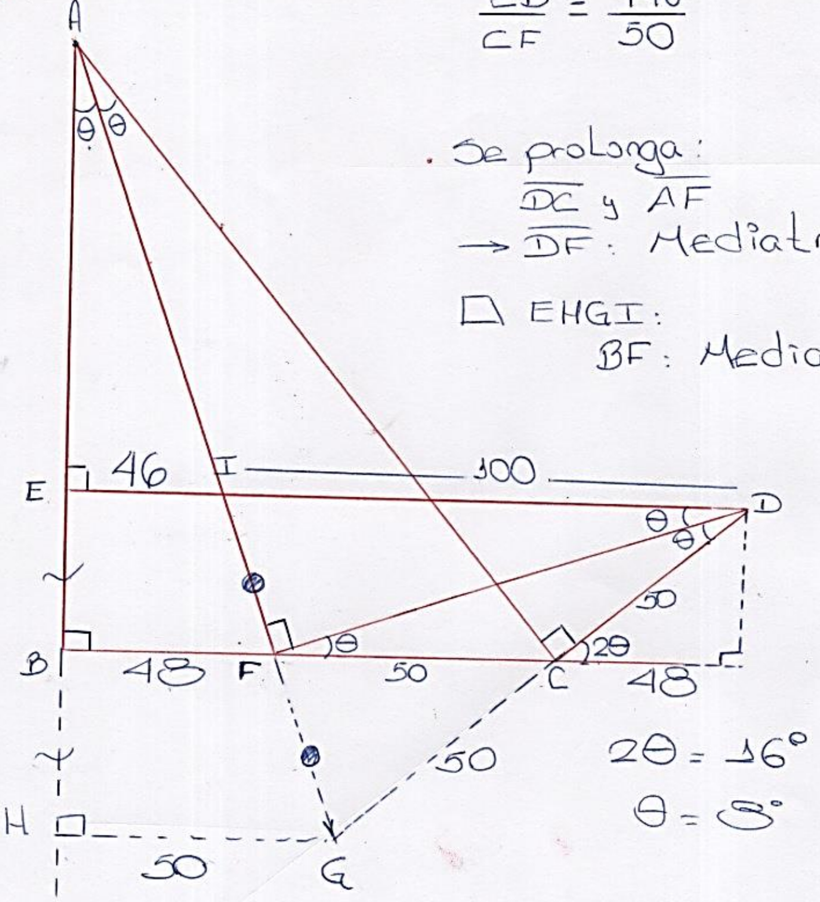
Calcular: $\mathrm{Tan}\theta + \sqrt{2}\mathrm{Sec}\theta$




A) $\frac{1}{7}$ B) $\frac{5}{7}$ C) $\frac{9}{7}$ D) $\frac{11}{7}$ E) $\frac{13}{7}$

12.

$$\frac{ED}{CF} = \frac{146}{50}$$

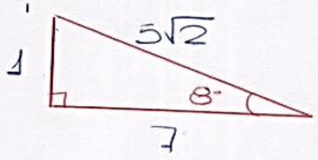



. Se prolonga:
$\overline{DC}$ y $\overline{AF}$
$\rightarrow \overline{DF}$: Mediatriz

⊿ EHGI:
BF: Mediana

$2\theta = 16°$
$\theta = 8°$

Calcular: $\tan\theta + \sqrt{2}\sec\theta$




$$\frac{1}{7} + \sqrt{2} \times \frac{5\sqrt{2}}{7}$$

$$\frac{11}{7}$$

CLAVE D

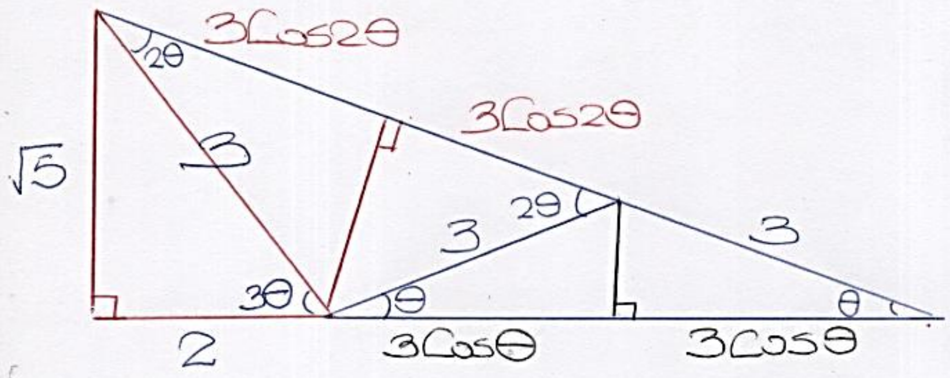
# Problema 13:

Siendo: $\text{Tan}3\theta = \frac{\sqrt{5}}{2}$, además $0 < \theta < 30°$.

Calcular: $P = \frac{1}{4}(1 + 2\text{Cos}2\theta)^2 - \frac{1}{9}(1 + 3\text{Cos}\theta)^2$

A) $\sqrt{5}$ B) $36$ C) $3$ D) $\frac{5}{36}$ E) $\frac{36}{5}$

13. $\tan 3\theta = \dfrac{\sqrt{5}}{2}$ ; $0 < \theta < 30° \xrightarrow{\times 3} 0 < 3\theta < 90°$




T. de Pitágoras

$$[3(1+2\cos 2\theta)]^2 = [2(1+3\cos\theta)]^2 + (\sqrt{5})^2$$

$$9(1+2\cos 2\theta)^2 = 4(1+3\cos\theta)^2 + 5$$

$$\frac{\cancel{9}(1+2\cos 2\theta)}{\cancel{36}} - \frac{\cancel{4}(1+3\cos\theta)}{\cancel{36}} = \frac{5}{36}$$

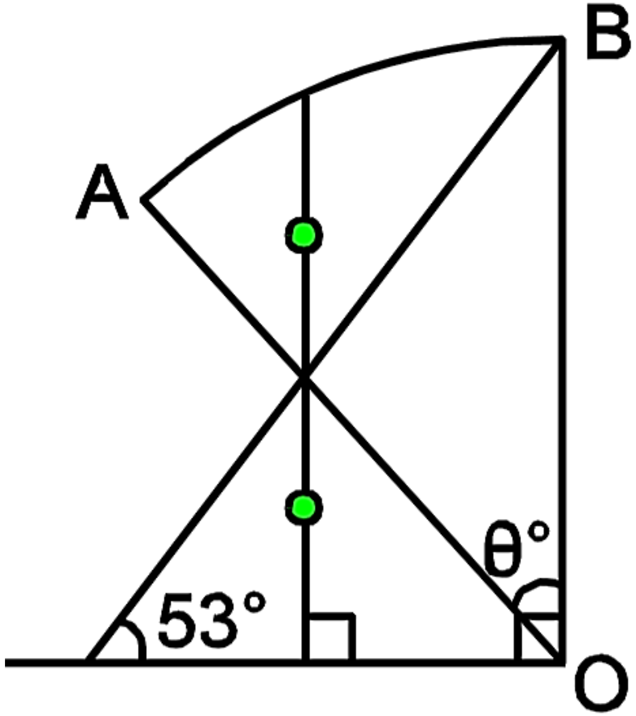
$$P = \frac{1}{4}(1+2\cos 2\theta)^2 - \frac{1}{9}(1+3\cos\theta)^2$$

$$\therefore P = 5/36$$

CLAVE D

# Problema 14:

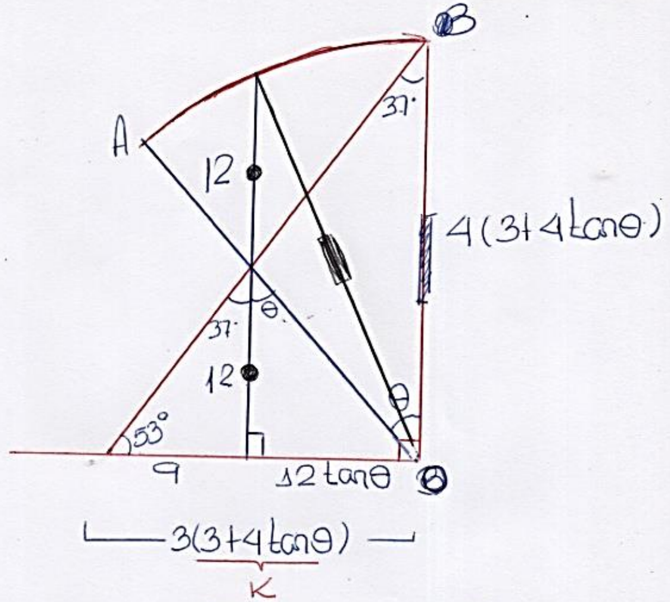
En la figura se muestra el sector circular AOB, del cual se pide calcular el valor de: $P = \sqrt{96\text{Tan}\theta + 28\text{Tan}^2\theta - 8}$




A) 7 B) 12 C) 10 D) 16 E) 9

14.-

$\sphericalangle AOB$: Sector Circular




T. Pitágoras:

$$[\cancel{4}(3+4\tan\theta)]^2 = (\cancel{4}\cdot 3\tan\theta)^2 + (\cancel{4}\cdot 6)^2$$

$$9 + 24\tan\theta + 16\tan^2\theta = 9\tan^2\theta + 36$$

$$24\tan\theta + 7\tan^2\theta = 27 \quad ) \times 4$$

$$96\tan\theta + 28\tan^2\theta = 108$$

$$P = \sqrt{96\tan\theta + 28\tan^2\theta - 8}$$

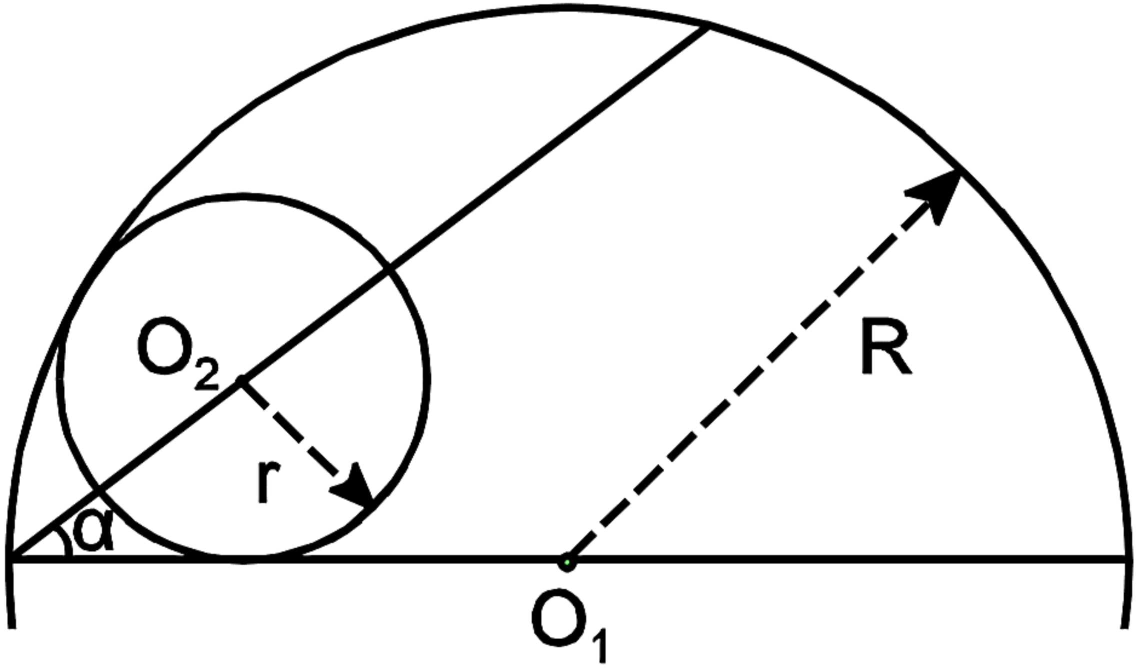
$$P = \sqrt{100}$$

$$\therefore P = 10$$

CLAVE C

# Problema 15:

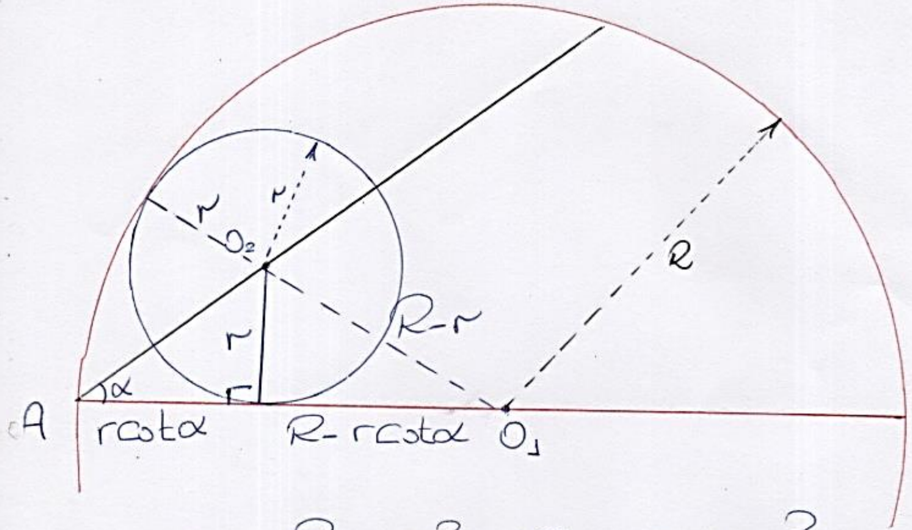
Dado el grafico mostrado, calcule: $\text{Tan}\alpha - \text{Tan}^2\alpha$ en función de r y R que son radios de las circunferencias con centros $O_2$ y $O_1$ respectivamente.




A) $2Rr$ B) $R^2 - r^2$ C) $\frac{r}{2R}$

D) $\frac{r^2}{2R}$ E) $\frac{R^2}{2r}$

15.- $\operatorname{Tan}\alpha - \operatorname{Tan}^2\alpha = ?$




$$(R-r)^2 = r^2 + (R - r\operatorname{Cot}\alpha)^2$$

$$\cancel{R^2} - 2Rr + \cancel{r^2} = \cancel{r^2} + \cancel{R^2} - 2Rr\operatorname{Cot}\alpha + r^2\operatorname{Cot}^2\alpha$$

$$2R\cancel{r}\operatorname{Cot}\alpha - 2R\cancel{r} = r^{\cancel{2}}\operatorname{Cot}^2\alpha$$

$\downarrow \tan^2\alpha$

$$2R\tan\alpha - 2R\tan^2\alpha = r$$

$$2R(\tan\alpha - \tan^2\alpha) = r$$

$$\therefore \tan\alpha - \tan^2\alpha = \frac{r}{2r}$$

CLAVE L

# Problema 16:

En un triangulo rectángulo ABC ( recto en C) se cumple:

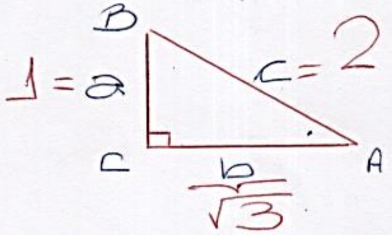
$$(\mathrm{SecB} - \mathrm{TanB})(\mathrm{CotA} + 2) = 1$$

Calcular :

$$\mathrm{Tan}\frac{\mathrm{C}}{2} + \mathrm{Tan}^2\mathrm{B} + 2\mathrm{SenA}$$

A) 5 B) 4 C) 3 D) 2 E) 1

16.-




$$c^2 = a^2 + b^2$$
$$a^2 = c^2 - b^2$$
$$a^2 = (c+b)(c-b)$$

$$(\operatorname{Sec}B - \operatorname{Tan}B)(\operatorname{Cot}A + 2) = 1$$
$$\left(\frac{c}{a} - \frac{b}{a}\right)\left(\frac{b}{a} + 2\right) = 1$$
$$\frac{(c-b)}{a} \cdot \frac{(b+2a)}{a} = 1$$
$$(c-b)(b+2a) = a^2$$

Reemp: $$\cancel{(c-b)}(\cancel{b} + 2a) = (c+\cancel{b})\cancel{(c-b)}$$
$$2a = 1c$$
$$\frac{a}{c} = \frac{1}{2}$$

Calcular: $$\operatorname{Tan}\left(\frac{C}{2}\right) + \operatorname{Tan}^2 B + 2\operatorname{Sen}A$$

$A = 30°$

$B = 60°$

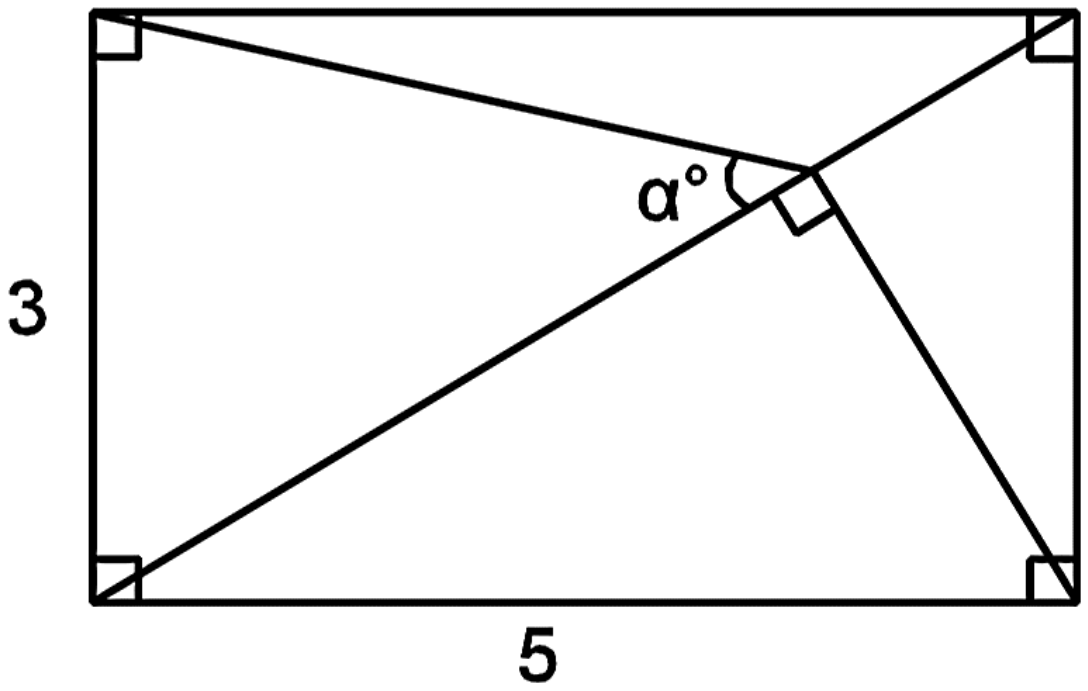
$C = 90°$

$$\operatorname{Tan}45° + \operatorname{Tan}^2 60° + 2\operatorname{Sen}30°$$
$$1 + (\sqrt{3})^2 + \cancel{2}\left(\frac{1}{\cancel{2}}\right)$$
$$5$$

CLAVE A

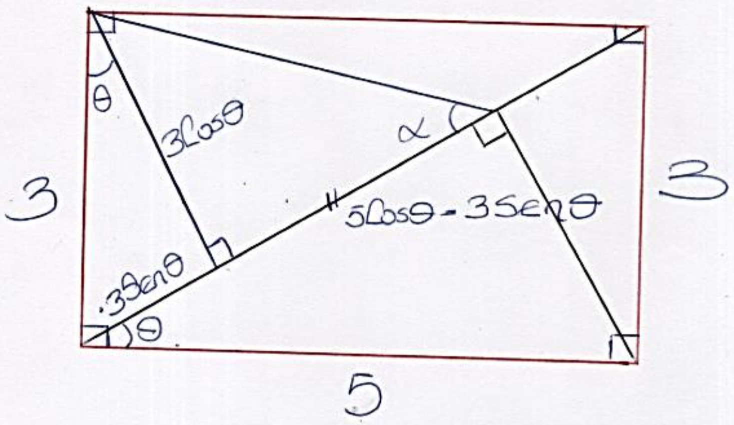
# Problema 17:

De la figura, calcular Tanα.




A) $\frac{21}{16}$ B) $\frac{16}{21}$ C) $\frac{15}{16}$ D) $\frac{13}{16}$ E) $\frac{14}{15}$

17.- $\tan\alpha = ?$




$$\tan\alpha = \frac{\frac{3\cos\theta}{\cos\theta}}{\frac{5\cos\theta}{\cos\theta} - \frac{3\operatorname{sen}\theta}{\cos\theta}}$$

$$\tan\alpha = \frac{3}{5 - 3\tan\theta}$$

$$\tan\alpha = \frac{3}{5 - 3 \cdot \frac{3}{5}}$$

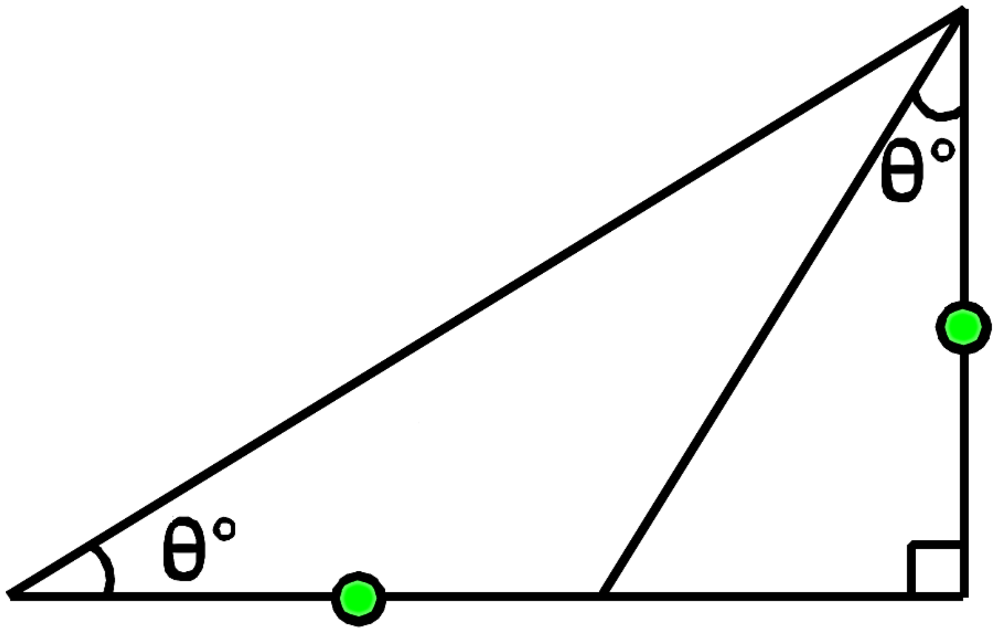
$$\therefore \tan\alpha = \frac{15}{16}$$

CLAVE C

# Problema 18:

Del grafico adjunto, calcular el valor de :

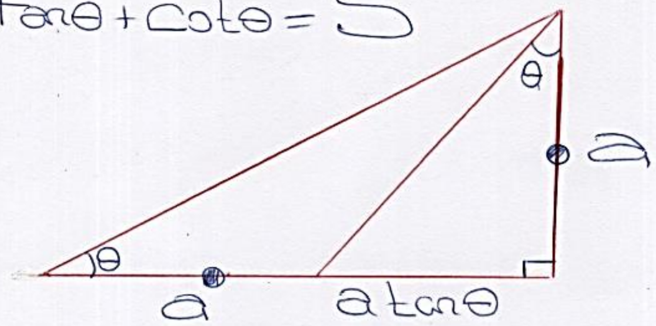
$$\operatorname{Tan}\theta + \operatorname{Cot}\theta$$




A) $\sqrt{7}$ B) $\sqrt{6}$ C) $\sqrt{5}$ D) 2 E) 1

18.-

$\tan\theta + \cot\theta = S$




$$\cot\theta = \frac{\cancel{a} + \cancel{a}\tan\theta}{\cancel{a}}$$

$$\cot\theta - \tan\theta = 1$$

Legendre:

$$(\cot\theta + \tan\theta)^2 - (\cot\theta - \tan\theta)^2 = 4\cot\theta\tan\theta$$

$$S^2 - 1^2 = 4(1)$$

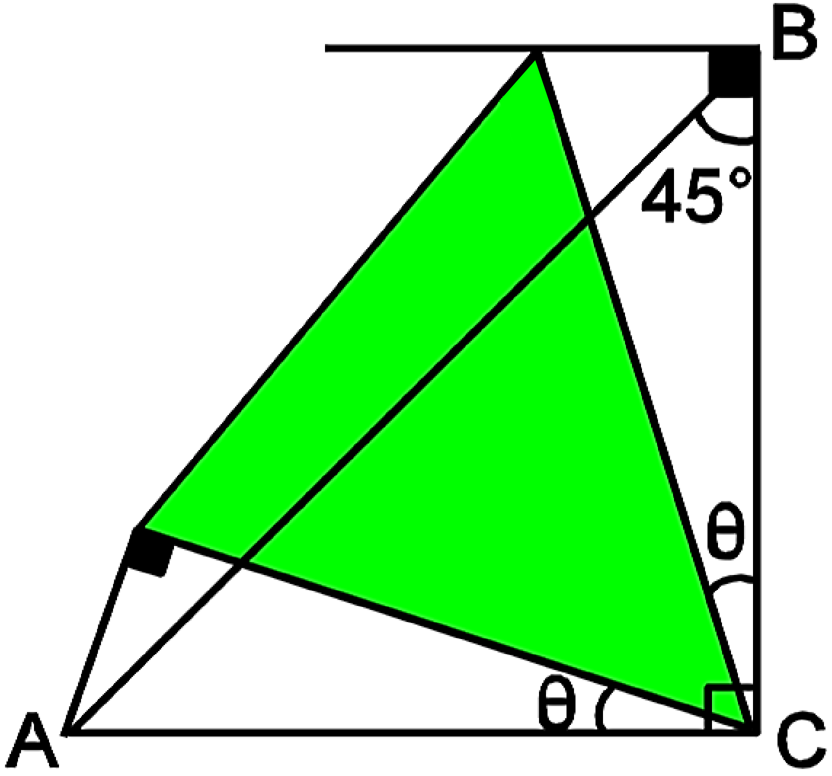
$$S^2 = 5$$

$$\therefore S = \sqrt{5}$$

CLAVE C

# Problema 19:

Halle el área de la región sombreada en la figura. Dato: $AB = 3\sqrt{2}$




A) $4{,}5\text{Tan}\theta$

B) $4{,}5\text{Cot}\theta$

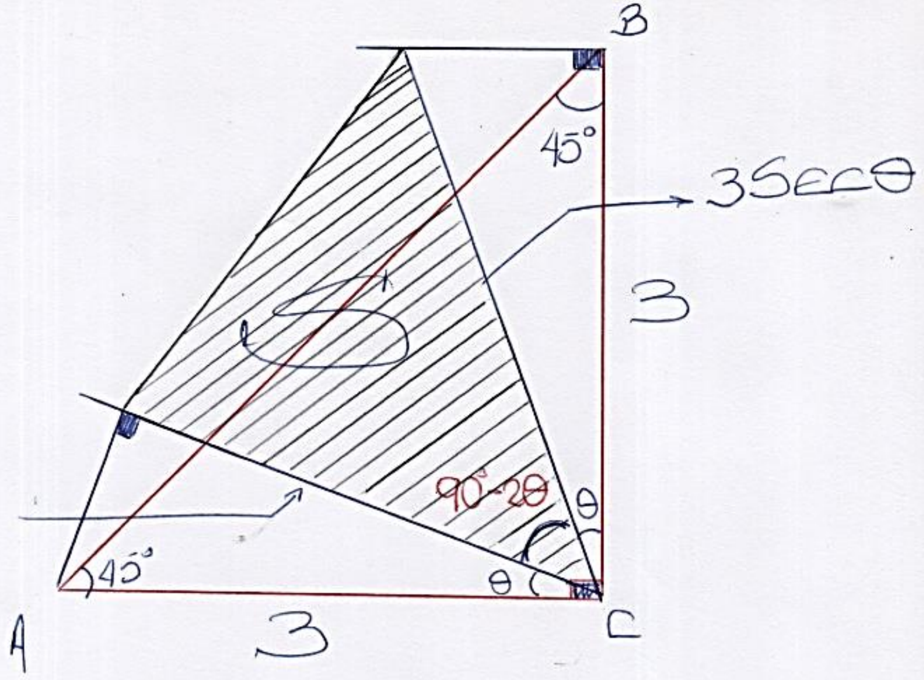
C) $4{,}5\text{Sen}\theta$

D) $4{,}5\text{Sen}2\theta$

E) $4{,}5\text{Cos}2\theta$

19.-

$AB = 3\sqrt{2}$



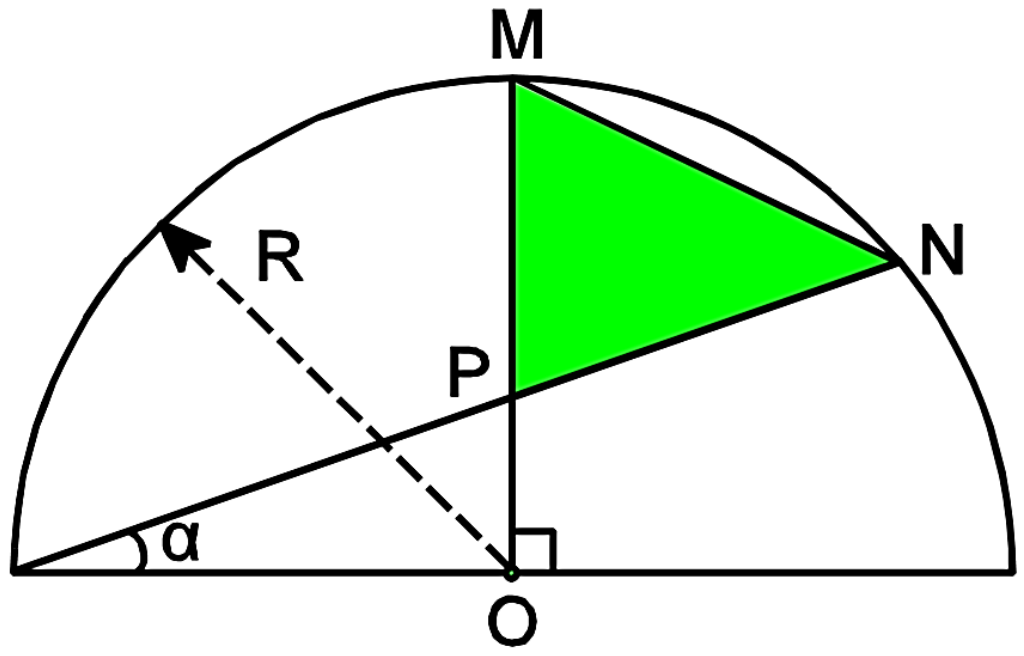

$$S = \frac{3\cos\theta \times 3\sec\theta}{2} \times \operatorname{Sen}(90° - 2\theta)$$

$$S = 4{,}5 \times 1 \times \underline{\cos 2\theta}$$

CLAVE $\underline{E}$

# Problema 20:

En la figura se muestra una semicircunferencia de radio R. Calcule el área de la región sombreada en función de α y R.




A) $R^2(\text{Cot}\alpha - 1)\text{Sen}2\alpha$

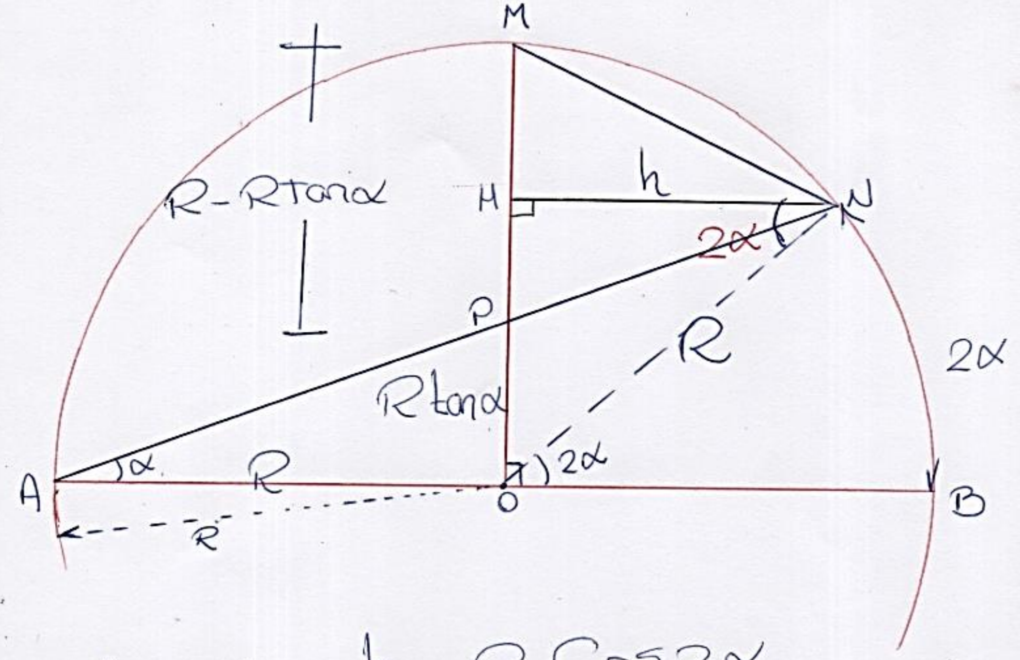
B) $R^2(1 - \text{Cot}\alpha)\text{Cos}2\alpha$

C) $0{,}5R^2(1 - \text{Tan}\alpha)\text{Cos}2\alpha$

D) $R^2(1 - \text{Cos}2\alpha)\text{Tan}\alpha$

E) $R^2(1 + \text{Tan}\alpha)\text{Sen}2\alpha$

20- $S_{\Delta PMN} = ?$




$\Delta NHO$ : $h = R\cos 2\alpha$

$\Rightarrow S = \dfrac{R(1-\tan\alpha)\times R\cos 2\alpha}{2}$

$S = 0{,}5 \cdot R^2 (1-\tan\alpha)\cos 2\alpha$ $\boxed{C}$

CLAVE $\boxed{C}$